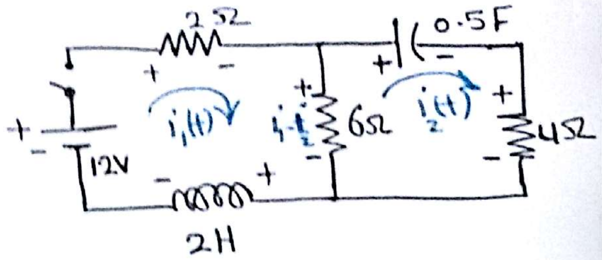
3- In the Following Circuit Find i_1, i_2 at $t = 0.2$, $t = 0.4$ using Euler's method.



Sol.

★ Apply Kirchhoff's Voltage law

$$12 = 2i_1 + 6(i_1 - i_2) + 2\frac{di_1}{dt}$$

$$\therefore \boxed{i_1' = -4i_1 + 3i_2 + 6} \quad \text{—— ①}$$

$$0 = 4i_2 + \frac{1}{0.5}\int i_2\,dt - 6(i_1 - i_2)$$

D.w.r.t

$$0 = 4i_2' + \frac{i_2}{0.5} - 6i_1' + 6i_2'$$

$$\therefore 10 i_2' = -2i_2 + 6i_1'$$

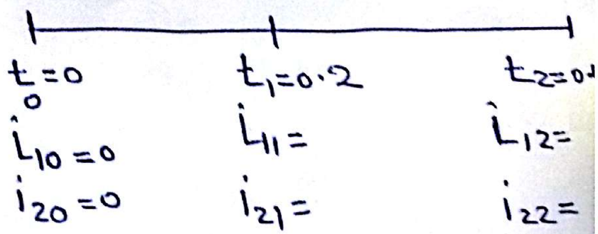
$$\therefore i_2' = -0.2 i_2 + 0.6 i_1'$$

From ①

$$i_2' = -0.2i_2 + 0.6(-4i_1 + 3i_2 + 6)$$

$$\boxed{i_2' = -2.4i_1 + 1.6i_2 + 3.6} \quad \text{—— ②}$$

★ Using Euler to Solve System



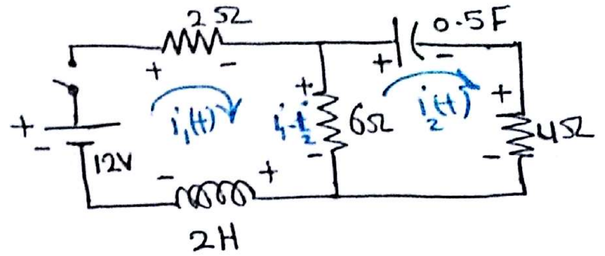
$$i_{11} = i_{10} + 0.2[-4i_{10} + 3i_{20} + 6]$$

$$i_{11} = 1.2\,A$$

$$i_{21} = i_{20} + 0.2[-2.4i_{10} + 1.6i_{20} + 3.6]$$

$$i_{21} = 0.72\,A$$

3- In the Following Circuit Find i_1, i_2 at $t=0.2$, $t=0.4$ using Euler's method.



Sol.

*Apply kirchhoff's Voltage law

$$12 = 2i_1 + 6(i_1 - i_2) + 2\frac{di_1}{dt}$$

$$\therefore \boxed{i_1' = -4i_1 + 3i_2 + 6} \quad — ①$$

$$0 = 4i_2 + \frac{1}{0.5}\int i_2\,dt - 6(i_1 - i_2)$$

D.w.r.t

$$0 = 4i_2' + \frac{i_2}{0.5} - 6i_1' + 6i_2'$$

$$\therefore 10i_2' = -2i_2 + 6i_1'$$

$$\therefore i_2' = -0.2i_2 + 0.6i_1'$$

From ①

$$i_2' = -0.2i_2 + 0.6(-4i_1 + 3i_2 + 6)$$

$$\boxed{i_2' = -2.4i_1 + 1.6i_2 + 3.6} \quad — ②$$

*Using Euler to Solve System

$t_0 = 0$: $i_{10} = 0$, $i_{20} = 0$

$t_1 = 0.2$: $i_{11} =$, $i_{21} =$

$t_2 = 0.4$: $i_{12} =$, $i_{22} =$

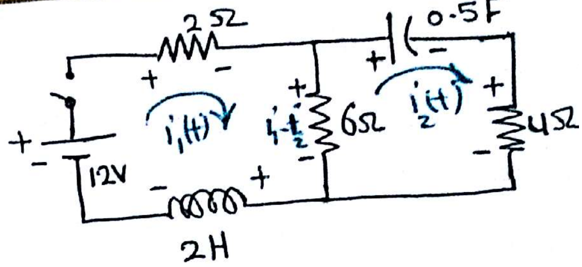
$$i_{11} = i_{10} + 0.2[-4i_{10} + 3i_{20} + 6]$$

$$i_{11} = 1.2A$$

$$i_{21} = i_{20} + 0.2[-2.4i_{10} + 1.6i_{20} + 3.6]$$

$$i_{21} = 0.72A$$

3- In the Following Circuit Find i_1, i_2 at $t=0.2, t=0.4$ using Euler's method.



Sol.

★Apply Kirchhoff's Voltage law

$$12 = 2i_1 + 6(i_1 - i_2) + 2\frac{di_1}{dt}$$

$$\therefore \boxed{i_1' = -4i_1 + 3i_2 + 6} \quad — (1)$$

$$0 = 4i_2 + \frac{1}{0.5}\int i_2\,dt - 6(i_1 - i_2)$$

D.w.r.t

$$0 = 4i_2' + \frac{i_2}{0.5} - 6i_1' + 6i_2'$$

$$\therefore 10 i_2' = -2i_2 + 6i_1'$$

$$\therefore i_2' = -0.2i_2 + 0.6i_1'$$

From (1)

$$i_2' = -0.2i_2 + 0.6(-4i_1 + 3i_2 + 6)$$

$$\boxed{i_2' = -2.4i_1 + 1.6i_2 + 3.6} \quad — (2)$$

★Using Euler to Solve System

$t_0 = 0$	$t_1 = 0.2$	$t_2 = 0.4$
$i_{10} = 0$	$i_{11} =$	$i_{12} =$
$i_{20} = 0$	$i_{21} =$	$i_{22} =$

$$i_{11} = i_{10} + 0.2[-4i_{10} + 3i_{20} + 6]$$

$$i_{11} = 1.2\,A$$

$$i_{21} = i_{20} + 0.2[-2.4i_{10} + 1.6i_{20} + 3.6]$$

$$i_{21} = 0.72\,A$$

Scanned by CamScanner

$i_{12} = i_{11} + 0.2[-4i_{11} + 3i_{21} + 6]$

$$i_{12} = 1.232A$$

$i_{22} = i_{21} + 0.2[-2.4i_{11} + 1.6\, i_{21} + 3.6]$

$$i_{22} = 1.0944A$$

4- Solve the 2^{nd} order initial-Value problem $y'' - 2y' + 2y = e^t \sin t$ For $0 \leq t \leq 0.2$ with $y(0) = -0.4$, $y'(0) = -0.6$. USing Euler's method with $h = 0.1$

Sol.

2^{nd} order $\Rightarrow$ System of 1^{st} order

let $y(t) = u_1(t)$, $y'(t) = u_2(t)$

$$\therefore \boxed{u_1' = u_2} \quad \text{— (1)}$$

$$u_2' = 2y' - 2y + e^t \sin t$$

$$\therefore \boxed{u_2' = -2u_1 + 2u_2 + e^t \sin t} \quad \text{— (2)}$$

*Using Euler with $h = 0.1$

$t_0 = 0$	$t_1 = 0.1$	$t_2 = 0.2$
$u_{10} = -0.4$	u_{11}	u_{12}
$u_{20} = -0.6$	u_{21}	u_{22}

$u_{11} = u_{10} + 0.1[u_{20}] = -0.46$

$u_{21} = u_{20} + 0.1[-2u_{10} + 2u_{20} + e^{t_0} \sin t_0]$

$= -0.64$

$u_{12} = u_{11} + 0.1[u_{21}] = -0.524$

$u_{22} = u_{21} + 0.1[-2u_{11} + 2u_{21} + e^{t_1} \sin t_1] = -0.7402$

• Adams - Bashforth Explicit method

$$y_{n+1} = y_n + h\left[f_n + \frac{1}{2}\nabla f_n + \frac{5}{12}\nabla^2 f_n + \frac{3}{8}\nabla^3 f_n + \cdots\right]$$

• القيم "y" قبل "y" المطلوبة يتم حسابها بـ "Euler"

• ex Approximate the solution to $y' = y - t^2 + 1$, $y(0) = 0.5$ Using Adams Bashforth at $y(0.9)$ with $h = 0.3$

Sol.

• Using Euler's method

$t_0 = 0$	$t_1 = 0.3$	$t_1 = 0.6$	$t_3 = 0.9$
$y_0 = 0.5$	$y_1 = ?$	$y_2 = ?$	$y_3 = ?$

$$y_1 = y_0 + h f(t_0, y_0)$$

$$y_1 = 0.95$$

$$y_2 = y_1 + h f(t_1, y_1)$$

$$y_2 = 1.508$$

t_i	f_i	∇f	$\nabla^2 f$
0	1.5		
0.3	$\frac{93}{50}$	$\frac{9}{25}$	
0.6	$\frac{537}{250}$	$\frac{36}{125}$	$\frac{-9}{125}$
0.9			

$$\therefore y_3 = y_2 + h\left[f_2 + \frac{1}{2}\nabla f_n + \frac{5}{12}\nabla^2 f_2\right]$$

$$\therefore y_3 = 2.1866$$

بسم الله الرحمن الرحيم

Numerical solution of P.D.E

الصورة العامة لمعادلة تفاضلية من الرتبة الثانية :

$$\boxed{Au_{xx}+Bu_{xy}+Cu_{yy}+Du_{x}+Eu_{y}+Fu+G=f(x,y)}$$

- **Elliptic**: $B^2-4AC<0$
- **Parabolic**: $B^2-4AC=0$
- **hyperbolic**: $B^2-4AC>0$

• لحل مسألة الـ P.D.E نتبع الخطوات التالية

• **Given:** المعادلة التفاضلية و الـ I.C و الـ B.C.

• **Req:** حل المعادلة والحصول على $u(x,y)$.

• **Sol.:** نظرا لصعوبة الحصول على "exact sol." للمعادلة فإننا نلجأ لإيجاد حلول تقريبية وذلك عن طريق تقسيم منطقة الحل إلى مجموعة من النقاط ونوجد قيمة "u" عند كل نقطة عن طريق التعويض في القانون ~~حسب~~ نوع المعادلة.

ملحوظة

لإثبات الصورة النهائية لكل نوع من المعادلات نستخدم القوانين :

• $u_{x_{i,j}} = \dfrac{u_{i+1,j}-u_{i,j}}{h}$ "or" • $u_{x_{i,j}} = \dfrac{u_{i,j}-u_{i-1,j}}{h}$

• $u_{x_{i,j}} = \dfrac{u_{i+1,j}-u_{i-1,j}}{2h}$ *

• $u_{xx_{i,j}} = \dfrac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{h^2}$

* نفس القوانين للمشتقات في الـ "y" حيث التغيير في "j".

• linear 2nd order P.D.E

$$A u_{xx} + B u_{xy} + C u_{yy} + D u_x + E u_y + F u = G$$

* Classification of P.D.E

$$B^2 - 4AC \Rightarrow \begin{cases} = 0 & \text{Parabolic} \rightarrow \text{heat Eq}^{\underline{n}}. \\ < 0 & \text{Elliptic} \rightarrow \text{laplace eq}^{\underline{n}}.\ (\text{poisson}) \\ > 0 & \text{hyperbolic} \rightarrow \text{wave eq}^{\underline{n}}. \end{cases}$$

• هنا يكون الحل على شكل شبكة "grid" حيث يوجد تغير في الـ "x" ، "t"

• Parabolic Eq$^{\underline{n}}$. "1-D heat Eq$^{\underline{n}}$."

$$u_t(x,t) = \alpha^2 u_{xx}(x,t)$$

$$u_t = \alpha^2 u_{xx}$$

$$u_x = \frac{u_{i+1,j} - u_{i,j}}{h} \quad \text{F.W}$$

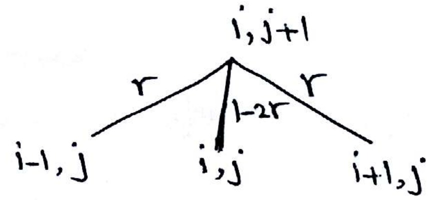
$$u_x = \frac{u_{i,j} - u_{i-1,j}}{h} \quad \text{B.W}$$

$$u_x = \frac{u_{i+1,j} - u_{i-1,j}}{2h} \quad \text{centeral}$$

$0 \leqslant x \leqslant L$, $t \geqslant 0$, $\underbrace{u(0,t) = u(l,t) = 0}_{\text{B.C}}$, $u(x,0) = f(x)$ I.C

• For Conditionally stable $0 \leqslant r \leqslant \frac{1}{2}$ $\Longrightarrow$ $r = \frac{k\alpha^2}{h^2}$

$$u_{i,j+1} = r u_{i-1,j} + (1-2r) u_{i,j} + r u_{i+1,j}$$



• في هذا النوع من المعادلات تكون الشبكة مفتوحة من فوق أي ليس لها نهاية.

*خطوات الحل

١- نقسم الشبكة في الـ x باستخدام الـ "h" وفي الـ "t" باستخدام الـ "K"

٢- نضع قيم الحل عند الحدود والـ initial

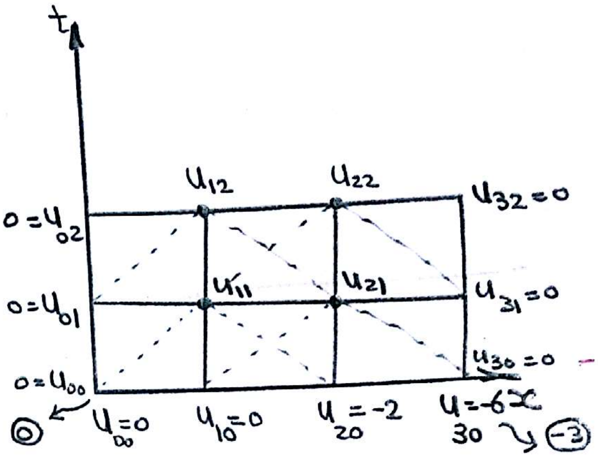
٣- نوجد القيم المطلوبة عند التقاطع.

1- Solve $u_t = u_{xx}$, $0 \leq x \leq 1$, $u(0,t) = u(1,t) = 0$,

$u(x,0) = 3x - 9x^2$ With $h = \frac{1}{3}$, $k = \frac{1}{18}$ Compute the 1st 2-rows

Sol.

$$r = \frac{k\alpha^2}{h^2} = \frac{1}{2} \Rightarrow 1 - 2r = 0$$



• 1st row

$$u_{11} = r u_{00} + r u_{20}$$

$$= \frac{1}{2}(0) + \frac{1}{2}(-2) = \boxed{-1}$$

$$u_{21} = r u_{10} + r u_{30}$$

$$= \frac{1}{2}(0) + \frac{1}{2}(-3) = \boxed{-1.5}$$

• 2nd row

$$u_{12} = r u_{01} + r u_{21}$$

$$= 0.5 * -1.5 = \boxed{-0.75}$$

$$u_{22} = r u_{11} + r u_{31}$$

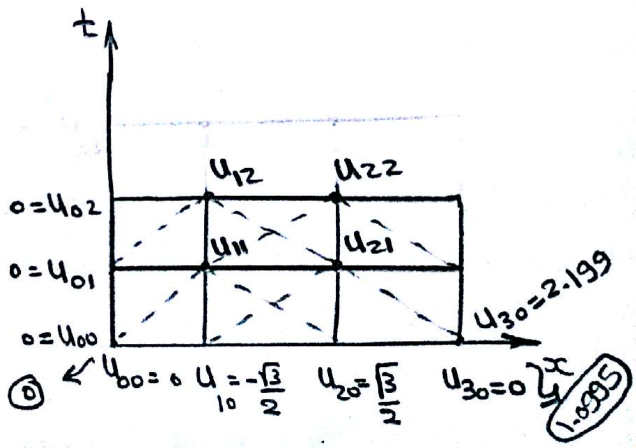
$$= 0.5 * -1 = \boxed{-0.5}$$

2- Solve $u_t = u_{xx}$, $0 \leq x \leq 2$, $u(x,0) = \sin 2\pi x$, $u(0,t) = 0$

$u_x(2,t) = 2e^{-4\pi^2 t}$ taking $h = \frac{2}{3}$, $k = \frac{2}{9}$

Sol.

$$r = \frac{\alpha^2 k^2}{h^2} = \frac{1}{2} \Rightarrow 1 - 2r = 0$$



I.C $u(x,0) = \sin 2\pi x$

$u_{00} = 0$

$u_{10} = -\frac{\sqrt{3}}{2}$

$u_{20} = \sqrt{3}/2$

• B.C $U_x(2,t) = 2e^{-4\pi^2 t_j}$

نختار (F.W or B.W or cen) هنا هختار قانون ال B.W عشان عارف النقطة اللي قبلها

$i-1,j \quad i,j \quad i+1,j$

$$U_{x_{i,j}} = \frac{U_{i,j} - U_{i-1,j}}{h}$$

$$\therefore U_{i,j} = hU_{x_{i,j}} + U_{i-1,j}$$

$$\therefore U_{30} = \frac{2}{3}\left(2e^{-4\pi^2 t}\right) + U_{20}\Big|_{t=0} = \frac{4}{3} + \frac{\sqrt{3}}{2} = 2.199$$

$$U_{31} = \frac{2}{3}\left(2e^{-4\pi^2 t}\right) + U_{21}\Big|_{t=\frac{2}{9}} = 2.06 * 10^{-3} + U_{21}$$

$$U_{32} = \frac{2}{3}\left(2e^{-4\pi^2 t}\right) + U_{22}\Big|_{t=\frac{4}{9}} = 3.2 * 10^{-8} + U_{22}$$

• 1st row

$$U_{11} = 0.5(U_{00} + U_{20}) = 0.5 * \frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{3}}{4}}$$

$$U_{21} = 0.5(U_{10} + U_{30}) = 0.5\left(-\frac{\sqrt{3}}{2} + 1.0995\right) = \boxed{0.117}$$

• 2nd row

$$U_{31} = 2.06 * 10^{-3} + 0.117 = \boxed{0.119}$$

$$\therefore U_{12} = 0.5(U_{01} + U_{21}) = 0.5 * 0.117 = \boxed{0.0585}$$

$$U_{22} = 0.5(U_{11} + U_{31}) = 0.5\left(\frac{\sqrt{3}}{4} + 0.119\right) = \boxed{0.276}$$

$$U_{32} = 3.2 * 10^{-8} + U_{22} \cong \boxed{0.276}$$